2023

Time - 3 hours

Full Marks - 80

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.
Candidates are required to answer in their own words as far as practicable.
The symbols used have their usual meanings.

GROUP – A

1. Answer all questions and fill in blanks as required. [1 × 12

(a) ________ has no reciprocal.

(b) inf $\mathbb{R}$ = ________.

(c) Total number of limit points of a finite set is ________.

(d) If (x_n) is monotonic decreasing and unbounded below, then $\lim_{n\to\infty} x_n$ = ________.

(e) If the series $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$ diverges to ∞, then what is the maximum value of α ?

(f) If $\sum a_n$ is absolutely convergent, then it is convergent. (True / False).

(g) $\lim_{x \to 0^-} \text{sgn}(x) =$ ________.

(h) A uniform continuous function is always continuous. (True / False)

(i) $\lim_{x \to 0^-} \frac{1}{|x|} = -\infty$. (True / False).

(j) Every discontinuous function is non-differentiable. (True / False)

(k) Give an example of a function which is continuous but not differentiable.

(l) Let f is a differentiable function on (a, b) such that $f'(x) < 0$ for all $x \in (a, b)$. Then what can you say about f(x) ?

GROUP – B

2. Answer any eight of the following questions. [2 × 8

(a) Write down the conditions under which a field F is said to be an ordered field.

(b) Show that N is unbounded above.

(c) Let $b \in \mathbb{R}_+$. Then prove that there exists $n \in N$ such that $\frac{1}{3^n} < b$.

(d) Test the convergence of $\sum \frac{10^n}{n!}$.

(e) Test the convergence of $\sum e^{-n^2}$.

(f) Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$.

(g) Prove that $\lim_{x \to \infty} \frac{1}{|x|} = 0$.

(h) Find $\lim_{x \to \infty} (x \sin \frac{1}{x})$.

(i) State Rolle's theorem.

(j) Let $f(x) = \begin{cases} x, & x \in Q \\ -x, & x \in \mathbb{R} - Q \end{cases}$. Show that $(f \circ f)(x) = x$.

GROUP – C

3. Answer <u>any eight</u> of the following questions. [3 × 8

(a) Let F be an ordered field and $a, b \in F$. Then show that $||a| - |b|| \le |a - b|$.

(b) Prove that if $a > 0$, then there exists $n \in N$ such that $\frac{1}{n} < a < n$.

(c) Show that the sequence $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots \frac{1}{n}$ is not Cauchy.

(d) Show that the series $\sum_{n=1}^{\infty} \frac{n+5}{n(n+1)\sqrt{n+2}}$ converges.

(e) Discuss the continuity of the function :

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & , x \neq 0 \\ 0 & , x = 0 \end{cases} \quad \text{at } x = 0.$$

(f) Let $f(x) < g(x)$ and $\lim_{x \to a} f(x) = L$, $\lim_{x \to a} g(x) = L'$.

Then show that $L \leq L'$.

(g) If f has limit, then show that it is unique.

(h) Prove that a function which is differentiable at a point is continuous at the point.

(i) Compute the approximate value of $\sqrt{17}$ using the principle of proportional part.

(j) Using the mean value theorem, show what $\tan^{-1} x < x$, $(x > 0)$.

GROUP – D

4. Answer any four of the following questions.

(a) State and prove the rational density theorem. [7

(b) Let X = $\mathbb{R}$ or $\mathbb{C}$. Then prove that every closed disk in X is a closed set in X. [7

(c) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^{\alpha}}$ converges if $\alpha > 1$ and diverges if $\alpha \leq 1$. [7

(d) Test the conditional and absolute convergence of the series :

$1-\left(\frac{1}{3}\right)+\left(\frac{1}{5}\right)-\left(\frac{1}{7}\right)+ \ldots\ldots\ldots.$ [7

(e) Prove that $\lim_{x\to a} f(x)=L$ if and only if

$\lim_{x\to a^+} f(x)=L=\lim_{x\to a^-} f(x).$ [7

(f) Prove that $f(x) = x^2$ is uniformly continuous on [a, b] but not on [a, ∞), a > 0. [7

(g) State and prove Caratheodory's theorem. [7

(h) State the prove Interior Extremum theorem. [7

No. of Printed Pages : 5 **Sem-II-Math-CC-4(R&B)**

2023

Time - 3 hours

Full Marks - 60

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in their own words as far as practicable.
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GROUP – A

1. Answer all questions and fill in blanks as required. [1 × 8

(a) Write the degree of the differential equation

$$\left(\frac{dy}{dt}\right)^3 = \sqrt{\left(\frac{dy}{dt}\right)^2 + 1}.$$

(b) The solution of the differential equation

$$x\left(y^2 - 1\right)dx - y\left(x^2 - 1\right)dy = 0.$$

(c) What is half life time of radioactive material uranium-238.

(d) Write the form of Bernoulli's equation.

(e) $M(x, y)dx + N(x, y)dy = 0$ is exact if and only if _________.

(f) If $y_1 = \sin x$, $y_2 = \cos x$, then $w[y_1, y_2] =$ ________.

(g) If the roots of the auxiliary equation of a homogeneous linear differential equation with constant coefficients are $1 \pm \sqrt{3}i$, then the general solution of the differential equation is ________.

(h) Write word equations of Influenza outbreak model.

GROUP – B

2. Answer any eight of the following questions. [1½ × 8

(a) Verify the equation $\left(y+\sqrt{x^2+y^2}\right)dx - x\,dy = 0$ is homogeneous or not.

(b) Find the general solution of the differential equation

$$y''' + 2y'' - y' - 2y = 0.$$

(c) Form the differential equation by eliminating the arbitrary constants of $y = at + be^t$.

(d) Write word equation for density dependent growth of population.

(e) Find C.F. of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 1 = 0$.

(f) Write the general form of Euler's equation of n^{th} order.

(g) Find the P.I. of $(D^2 + 1)y = \cos x$.

(h) Write the U.C. sets of U.C. function $f(x) = x^3$.

(i) Find the equilibrium points of the coupled system $\frac{dx}{dt} = y$, $\frac{dy}{dt} = -x$.

(j) Write the balanced equation in word form for the limited growth with harvesting.

GROUP – C

3. Answer <u>any eight</u> of the following questions. [2 × 8

(a) Find the integrating factor of the differential equation

$$\frac{dy}{dx} + y \tan x = \sin 2x.$$

(b) Solve $(\cos x + y \sin x)dx = \cos x\, dy$, $y(\pi) = 0$.

(c) Solve the differential equation $xy\, dx + (x + 1)dy = 0$.

(d) Find an expression for the time for the population of the average life expectancy.

(e) Discuss equilibrium solution and stability of the model having differential equation $\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right)$.

(f) Find the general solution of the differential equation

$$y'' - 4y' + 4y = 0.$$

(g) Find the particular integral of $(D-2)^2 y = e^{2x}$.

(h) Solve $(D^2 - 2D + 1)y = e^{-x}$.

(i) Determine phase plane trajectory directions of predator-prey model.

(j) Formulate the differential equation of single battle model.

GROUP – D

4. Answer any four of the following questions.

(a) Solve : $x^2 \frac{dy}{dx} + (1-2x)y - x^2 = 0$. [6

(b) Solve : $x\frac{dy}{dx} = y + 2x\, e^{\frac{-y}{x}}$. [6

(c) Taking production rate 1, carrying capacity as 10, harvesting rate $\frac{9}{10}$ and initial population x_0, write the differential equation of the limited growth and harvesting model to solve it and interpret the solution. [6

(d) Find a differential equation for the amount of salt in the tank at any time, in Lake pollution model. [6

(e) Find the particular integral of $(D^2 + 2)y = x^2 e^{3x}$. [6

(f) Find the general solution of the equation $(D^2 + 1)y = \sec x$ by the method of variation of parameter. [6

(g) Find the equilibrium solutions of the differential equation of predator-prey model. [6

2023

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GROUP – A

1. Fill in the blanks. (all) [1 × 12

(a) The curvature at any point of a circle is ________.

(b) Area of a loop of the curve $ay^2 = x^2(a - x)$ is ________.

(c) The number of loops in the curve $r = a \sin \theta$ is ________.

(d) If one end of a diameter of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 7 = 0$ is $(-1, 2, 4)$, then the other end is ________.

(e) $\lim_{x \to 0} \frac{\sin 5x}{\tan 3x} =$ ________.

(f) Let $f(x)=\frac{\sin x}{x}$. If f(x) becomes continuous at $x = 0$, then ________ is the value of f(0).

(g) The value of the derivative of $|x-1|+|x-3|$ at $x = 2$ is ________.

(h) If $f(x, y) = x^3 + y^3 - 2x^2y^2$, then $(f_{xx})_{x=y=1}$ is ________.

(i) If $u = f(y-z, z-x, x-y)$, then the value of

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\_\_\_\_\_\_\_\_.$$

(j) ________ and ________ is the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}-\left(\frac{dy}{dx}\right)^{\frac{1}{2}}-4=0$ respectively.

(k) ________ is the condition the differential equation Mdx + Ndy = 0 is exact.

(l) ________ is the number of arbitrary constants in general solution of a first order and n^{th} degree differential equation.

GROUP – B

2. Answer <u>any eight</u> of the following questions. [2 × 8

(a) Define radius of curvature for Cartesian curve.

(b) Find the asymptotes parallel to the coordinate axes of the curve $(x^2 + y^2)x - ay^2 = 0$.

(c) Find the centre and the radius of the sphere whose equation is given by $x^2 + y^2 + z^2 - 8x - 6y - 10z = 0$.

(d) What is the Maclaurin's series expansion of e^x?

(e) Verify Rolle's theorem for the function f defined by

$$f(x) = x^2 - 3x + 2 \text{ on the interval } [1, 2].$$

(f) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(g) Evaluate : $\lim_{x \to 0} x \operatorname{cosec} x$.

(h) Solve : $x(y^2 - 1)dx - y(x^2 - 1)dy = 0$.

(i) Solve : $p^2 + y^2 = 4$.

(j) Find P.I. of the differential equation $(D^2 + D + 1)y = \sin 2x$.

GROUP – C

3. Answer any eight of the following questions. [3 × 8

(a) Trace the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$.

(b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(c) Find the asymptotes of the curve $x^2y + xy^2 + xy + y^2 + 3x = 0$.

(d) Find the equation to the sphere through the points (0, 0, 0), (0, 1, –1), (–1, 2, 0) and (1, 2, 3).

(e) Using Lagrange's mean value theorem, prove that $\sin x < x$ in the interval $(0, \pi/2)$.

(f) Evaluate $\lim_{x \to 0} (1+x)^{1/x}$.

(g) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, $x \neq y$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

(h) Find the maxima and minima of the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$

(i) Solve : $\frac{dy}{dx} + y = e^{-x}$.

(j) Solve : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = e^x$.

GROUP – D

4. Answer any four of the following questions.

(a) Find the equation of the cone whose vertex is (1, 1, 0) and guiding curve is $x^2 + z^2 = 4$, $y = 0$. [7

(b) Find the length of the entire circle $x^2 + y^2 = 2ax$. [7

(c) Let $f(x, y) = \begin{cases} \dfrac{x^2y^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$ [7

Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$.

(d) Find the maximum value of $x^4 + y^4 + z^4$ subject to $xyz = c^2$. [7

(e) Expand $\log(\sin x)$ in powers of $(x - 2)$ by Taylor's Theorem. [7

(f) Solve : $x = y + a \log p$. [7

(g) Find a particular integral of $(D^2 + 1)y = \sec x$ by using method of variation of parameter. [7